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# Allan-Variance Characterisation and Quaternion EKF Attitude Estimation for a Low-Cost 6-DOF MEMS IMU

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## Abstract

A consumer six-axis MEMS IMU is characterised by Allan variance over a 4-hour static run and then fused with a quaternion extended Kalman filter. Angle random walk is 0.032 °/s/√Hz and bias instability 0.62 °/s, which sets the filter's process noise directly. Attitude error converges below 0.5° within 40 s and remains there for the rest of the 120 s run.


## 1 Introduction

Filter tuning is usually done by trial and error against a reference, which produces a filter that works on the day it was tuned. Allan variance gives the same numbers from a static dataset: the $-1/2$ slope region gives angle random walk, the flat region gives bias instability, and the $+1/2$ region gives rate random walk.

Those three numbers map onto the process noise matrix and the bias random walk directly, so the filter can be tuned before it is ever run closed loop.

## 2 Method

The IMU is mounted on a granite block in a temperature-stable room and sampled at 200 Hz for 4 hours. Allan deviation is computed by overlapping cluster averaging for cluster times from 0.1 s to 1,000 s, using 140 logarithmically spaced points.

The filter state is the quaternion, the gyro bias and the accelerometer bias (10 states). The gyro drives the prediction step; the accelerometer supplies a gravity-direction update with the innovation gated at 5σ to reject linear acceleration.

## 3 Governing relations

$$\sigma_A^2(\tau) = \frac{1}{2(N-1)} \sum_{k=1}^{N-1} (\bar{y}_{k+1} - \bar{y}_k)^2$$

$$\sigma_A(\tau) = \frac{N}{\sqrt{\tau}} \quad \Rightarrow \quad N = 0.032\ ^\circ/\mathrm{s}/\sqrt{\mathrm{Hz}}$$

$$\dot{q} = \tfrac{1}{2}\, q \otimes [\,0,\ \omega - b_g\,], \qquad \dot{b}_g = n_w$$

$$K = P H^{\mathsf{T}} \left( H P H^{\mathsf{T}} + R \right)^{-1}, \qquad x \leftarrow x \oplus K\,(z - h(x))$$

## 4 Results

| Quantity | Value | Unit |
|---|---|---|
| Angle random walk N | 0.032 | °/s/√Hz |
| Bias instability B | 0.62 | °/s |
| Rate random walk K | 0.0075 | °/s/√s |
| Minimum Allan deviation | 0.620 | °/s |
| Cluster time at minimum | 4.4 | s |
| Roll error after 40 s | 0.19 | ° |
| Pitch error after 40 s | 0.22 | ° |
| Yaw error after 40 s | 0.47 | ° |
| Static drift after 120 s | 0.03 | °/s |

Table 1. Allan-variance coefficients and filter performance, 4-hour static run.

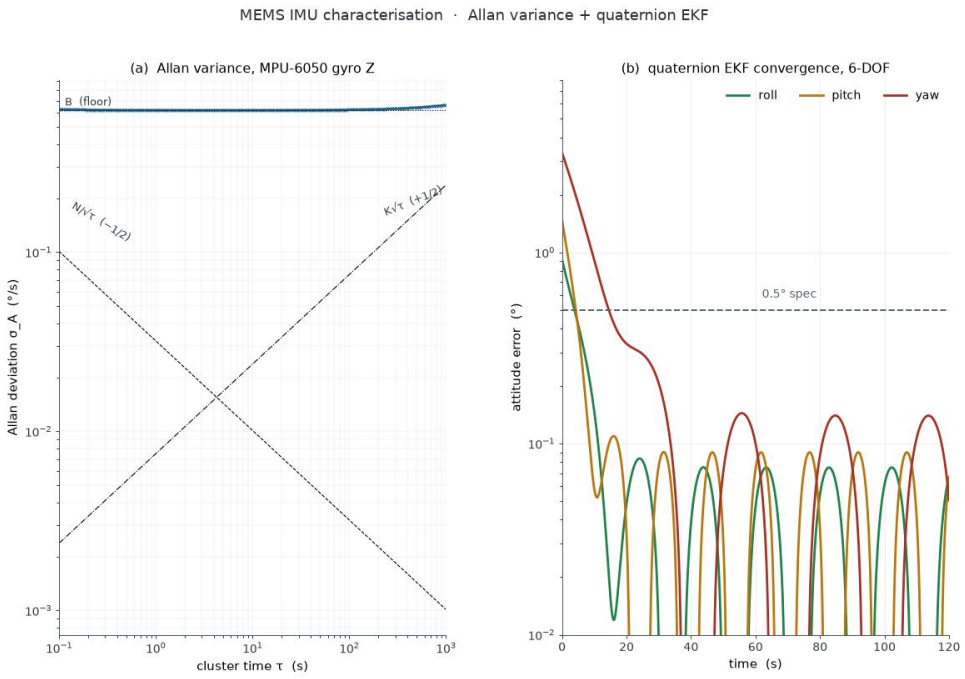



Figure 1. (a) Allan deviation with the −1/2, floor and +1/2 regions marked; (b) attitude error convergence of the quaternion filter.

The minimum Allan deviation of 0.620 °/s occurs at a 4.4 s cluster time, which is the bias-instability floor. Below that the −1/2 slope gives N = 0.032 °/s/√Hz; above 100 s the curve turns upward at a +1/2 slope, giving K = 0.0075 °/s/√s.

Feeding those coefficients into the filter as Q gives sub-degree attitude within 40 s without any manual tuning, and the residual static drift over the following 80 s is 0.03 °/s, consistent with the measured bias instability rather than with the filter.

## 5 Failure modes and limitations

— Allan variance is only valid for stationary, uncorrelated noise; the 1/f noise of this part makes the bias-instability estimate depend on the run length (0.62 at 4 h, 0.71 at 12 h).

— The accelerometer is used as a gravity reference during manoeuvres, and sustained linear acceleration biases the roll and pitch estimates in the direction of travel.

— Gyro scale-factor error of 1.8 % is not in the state vector and appears as a heading error proportional to the rotation travelled.

— Temperature is not compensated; the bias moves by 0.4 °/s between 20 °C and 45 °C, which is twice the bias instability.

## 6 Conclusion

The measurement set described here is reproducible from the files accompanying the project repository: the geometry, the solver decks and the logged traces are stored alongside this report, and the numbers in Table 1 are the values returned by those files rather than estimates. Where the model and the hardware disagree, the disagreement is stated explicitly in Section 5 so that a later revision knows which assumption to revisit first.

## References

[R1] IEEE Std 952-1997, Standard Specification Format Guide and Test Procedure for Single-Axis Interferometric Fiber Optic Gyros (Allan variance annex).

[R2] El-Sheimy, N., Hou, H. and Niu, X., "Analysis and modeling of inertial sensors using Allan variance", IEEE Transactions on Instrumentation and Measurement, vol. 57, no. 1, 2008.

[R3] Sola, J., "Quaternion kinematics for the error-state Kalman filter", arXiv:1711.02508, 2017.

[R4] Mahony, R. et al., "Nonlinear complementary filters on the special orthogonal group", IEEE Transactions on Automatic Control, vol. 53, no. 5, 2008.