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Closed-Form Inverse Kinematics and Stall-Current Detection for a Four-Degree-of-Freedom Serial Manipulator

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Abstract

A four-axis serial arm with a 680 mm reach is characterised end to end. Inverse kinematics is solved in closed form with a shoulder-elbow-wrist decomposition and verified against a damped-least-squares solver; the two agree to 0.1 mm over 10,000 randomly sampled targets. Wrist stall is detected from the commutation current 38 ms before the encoder loses count, using a 4.8 A threshold against a 1.9 A nominal draw.

1 Introduction

A four-axis arm — base yaw, shoulder pitch, elbow pitch, wrist pitch — is the smallest kinematic chain that can place a tool at an arbitrary position and orientation within a vertical plane through the base axis. Its inverse kinematics has a closed form, which matters on a 200 Hz control loop where an iterative solver would consume the cycle budget.

The second half of the problem is not geometric. The wrist carries the least torque and the most friction, so it is where a grasp jams first. Because the drive is sensorless, the only evidence is the commutation current, and the useful question is how early that evidence appears relative to the encoder.

2 Method

The base yaw θ1 is obtained from the target azimuth. The remaining three joints are solved as a planar two-link problem with a rigid wrist offset, giving two elbow solutions; the branch is selected by continuity with the previous command. Joint limits are enforced by rejecting the unreachable branch rather than by clamping, which avoids the wrist flipping through a singularity.

Stall detection compares the low-pass filtered commutation current against a velocity-dependent threshold. The filter is a second-order Butterworth at 220 Hz, and the threshold is 2.5× the current predicted by the static gravity model at the measured pose.

3 Governing relations

$$\theta_1 = \operatorname{atan2}(y, x), \quad r = \sqrt{x^2 + y^2}$$

$$r^2 = a_1^2 + a_2^2 + 2 a_1 a_2 \cos\theta_3 \quad \Rightarrow \quad \theta_3 = \pm \arccos\left(\frac{r^2 - a_1^2 - a_2^2}{2 a_1 a_2}\right)$$

$$\theta_2 = \operatorname{atan2}(r, z) - \operatorname{atan2}(a_2 \sin\theta_3, a_1 + a_2 \cos\theta_3)$$

$$\tau = J^{\mathsf{T}} F_g, \quad F_g = \sum_k m_k g \quad \text{(static gravity load)}$$

4 Results

Quantity	Closed form	DLS (200 it.)	Unit
Mean position error	0.04	0.09	mm
Maximum position error	0.10	0.34	mm
Solve time per pose	3.1	412	µs
Branch-flip events	0	7	—
Reach (max)	680	680	mm
Payload at full reach	1.2	—	kg
Stall threshold	4.8	—	A
Nominal draw at 0.5 m/s	1.9	—	A
Detection lead time	38	—	ms

Table 1. Arm characterisation, 10,000 random reachable targets.



Figure 1. Four-axis arm on the plywood test base. The breadboard behind the arm carries the driver wiring and the current-sense tap used for the stall tests of Table 1.

The closed-form solution returns a mean position error of 0.04 mm and a worst case of 0.10 mm, against 0.09 mm and 0.34 mm for damped least squares run to 200 iterations. The closed form is 130× faster, which is what allows the 200 Hz loop to also run the current filter and the trajectory interpolator in the same cycle.

Stall detection fires 38 ms before the encoder loses count at 0.5 m/s approach speed. The margin is comfortable at low speed but falls to 9 ms at 1.2 m/s, so the threshold is scheduled with velocity rather than fixed.

5 Failure modes and limitations

— The wrist offset is treated as rigid; the measured 0.35° backlash at the wrist gearbox produces a 4.1 mm tool-position error that the kinematics does not model.

— The gravity model assumes point masses at link midpoints; the true inertia distribution biases the shoulder torque by 6 % and therefore the stall threshold by the same amount.

— At the fully extended pose the Jacobian is ill-conditioned and the closed form is numerically sensitive to the 0.1 mm encoder resolution.

— Current-based stall detection cannot distinguish a jam from a payload at the top of the rated range without an additional velocity check.

6 Conclusion

The measurement set described here is reproducible from the files accompanying the project repository: the geometry, the solver decks and the logged traces are stored alongside this report, and the numbers in Table 1 are the values returned by those files rather than estimates. Where the model and the hardware disagree, the disagreement is stated explicitly in Section 5 so that a later revision knows which assumption to revisit first.

References

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