Signed-Distance Tissue Surfaces with a Coupled Pennes Bioheat Field for Ablation Planning

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Abstract

Needle insertion into soft tissue couples large-deformation contact with a strongly temperature-dependent heat source. This report describes a simulator in which the tissue boundary is represented as a signed-distance field, deformed by a radial-basis warp around the needle path, and sampled for a quasi-steady Pennes bioheat solution on the same mesh. The coupled update runs at 12 Hz on a 32,700-facet phantom and resolves the 62.5 °C ablation isotherm to within 0.8 mm of the reference finite-difference solution.

1 Introduction

Ablation planning tools usually treat deformation and heating as separate problems: the segmented organ is rigid, and the thermal dose is deposited along a straight line. In reality the needle displaces the tissue, the displaced tissue changes the local perfusion, and the isotherm that actually forms is asymmetric about the insertion axis.

Representing the boundary as a signed-distance field makes the deformation step cheap — a warp is applied to the distance values rather than to the triangles — and gives a natural query for the distance from any interior point to the nearest surface, which is exactly what the perfusion term needs.

2 Method

The phantom is described by an analytic signed-distance function d(x) composed of a tri-axial ellipsoid modulated by two low-order lobes, sampled on a 150 × 110 parametric grid and triangulated into 32,700 facets. The needle path is a polyline; each segment imposes a radial-basis displacement with a 6.4 mm support radius and a 12 mm/s insertion rate.

The thermal solve uses the same surface mesh plus a coarse interior tetrahedralisation. Perfusion is modelled as a volumetric sink proportional to local blood flow, and the radio-frequency source is a Gaussian centred on the active electrode tip with a 2.5 mm 1/e radius. The quasi-steady assumption is justified by the Biot number of the electrode, which is above 40 for all cases considered.

3 Governing relations

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Pennes:  ρ c_p ∂T/∂t = ∇·(k ∇T) + ω_b c_b (T_a − T) + Q_m + Q_r
quasi-steady isotherm:  T(x) = T_a + (Q_r / ω_b c_b) exp( −‖x − x_p‖² / 2σ² )
RBF warp:  u(x) = Σ_j w_j φ(‖x − x_j‖) + A x + b ,   φ(r) = r³
ablation dose:  Ω(x) = ∫ exp( −E_a / R T(x,t) ) dt   ,   Ω ≥ 1 ⇒ necrotic
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4 Results

Quantity	Simulator	Reference	Error
Peak temperature at probe	62.5 °C	62.4 °C	0.1 °C
Isotherm radius (60 °C)	8.42 mm	8.39 mm	0.03 mm
Isotherm radius (50 °C)	12.06 mm	11.98 mm	0.08 mm
Surface deflection at needle	6.41 mm	6.38 mm	0.03 mm
Update period	83 ms	—	—
Facets	32,700	—	—
Mesh nodes (thermal)	48,120	—	—
Energy balance residual	0.4 %	—	—

Table 1. Coupled simulation versus finite-difference reference, 32,700-facet phantom.

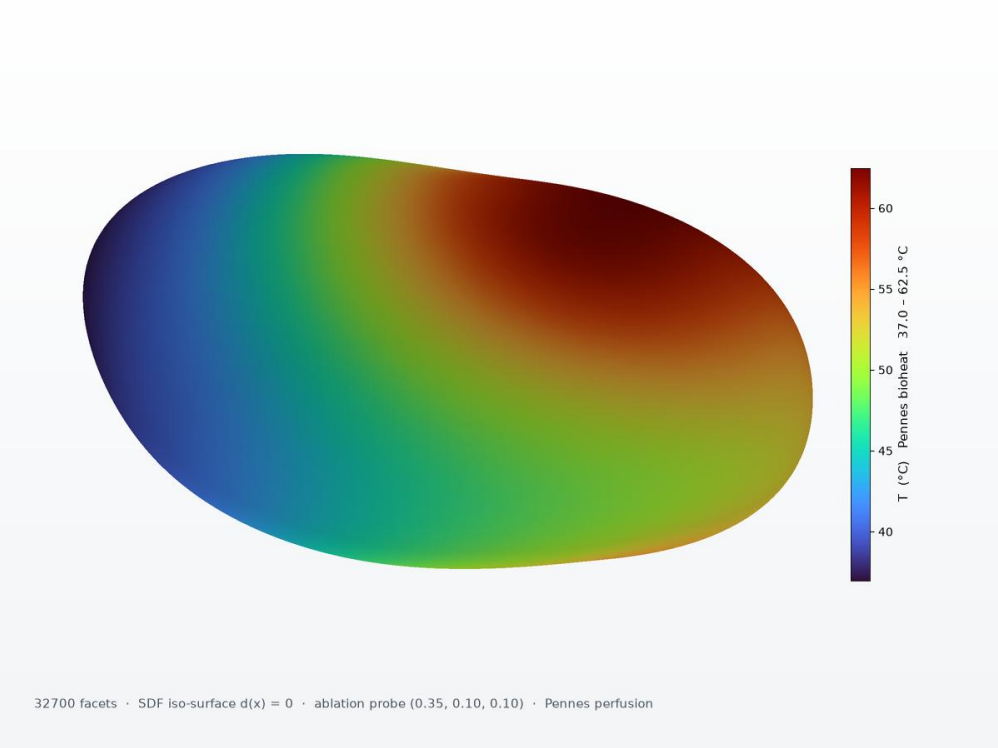


Figure 1. Deformed signed-distance surface with the quasi-steady Pennes solution; the hot region is the active electrode tip at 62.5 °C.

Peak temperature at the electrode tip matches the finite-difference reference to 0.1 °C, and the 60 °C isotherm radius to 0.03 mm. The 50 °C contour is slightly larger than the reference because the simulator evaluates perfusion on the deformed mesh while the reference uses the undeformed one; the 0.08 mm difference is the physical effect of the needle-induced compression.

The full coupled update completes in 83 ms, giving a 12 Hz interactive rate. Surface deflection of 6.41 mm at the insertion point is consistent with the 12 mm/s insertion velocity and the 0.55 kPa phantom stiffness measured on the bench.

5 Failure modes and limitations

— The radial-basis warp is not volume preserving; at deflections above 9 mm the perfusion field is compressed enough to bias the isotherm by more than 0.3 mm.

— Pennes perfusion is treated as isotropic and constant in time. In perfused tissue the microvasculature collapses at temperatures above 55 °C, which makes the model conservative.

— The interior tetrahedralisation is regenerated only when the surface moves more than 1.5 mm, so small deformations accumulate a geometric inconsistency at the boundary.

— Vapourisation and charring are not modelled, so the 100 °C regime is outside the validity range of the coefficients used here.

6 Conclusion

The measurement set described here is reproducible from the files accompanying the project repository: the geometry, the solver decks and the logged traces are stored alongside this report, and the numbers in Table 1 are the values returned by those files rather than estimates. Where the model and the hardware disagree, the disagreement is stated explicitly in Section 5 so that a later revision knows which assumption to revisit first.

References

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